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ALL INDIA TEST SERIES CSE-2023

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4. SUBJECT:- Nuclear and Particle Physics
5. DATE:- 14-07-2024

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Q.NO	MARKS
1.	30½
2.	33
3.	30
4.	33
5.	30½
6.	
7.	
8.	

TOTAL MARKS	157/250

Excellent

Keep it up!

Don

EXAMINER SIGNATURE

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Ans 1(a) Nuclear Isomerism – The delayed decay of a radioactive isotope because of existence of different metastable energy states is known as nuclear isomerism.

The states have different half-lives

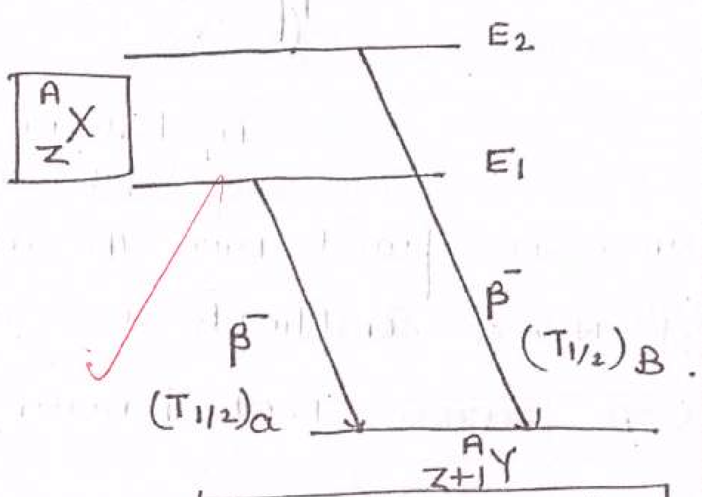


Fig. Nuclear Isomerism

(Explanation) – Shell model explains it with help of shell structure of nucleus. When a high spin state is depressed close to low spin state opposite in parity, the emitted γ-ray during transition from High to Low spin state leads to high long life.

Eg $1f_{7/2}$ with $1d_{3/2}$

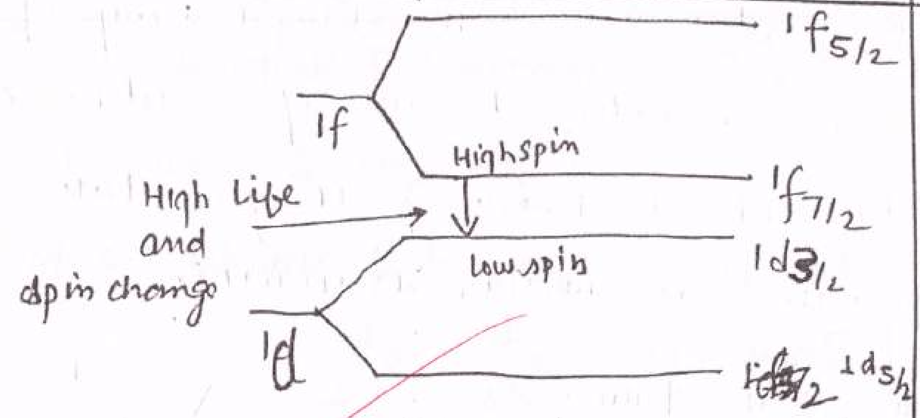
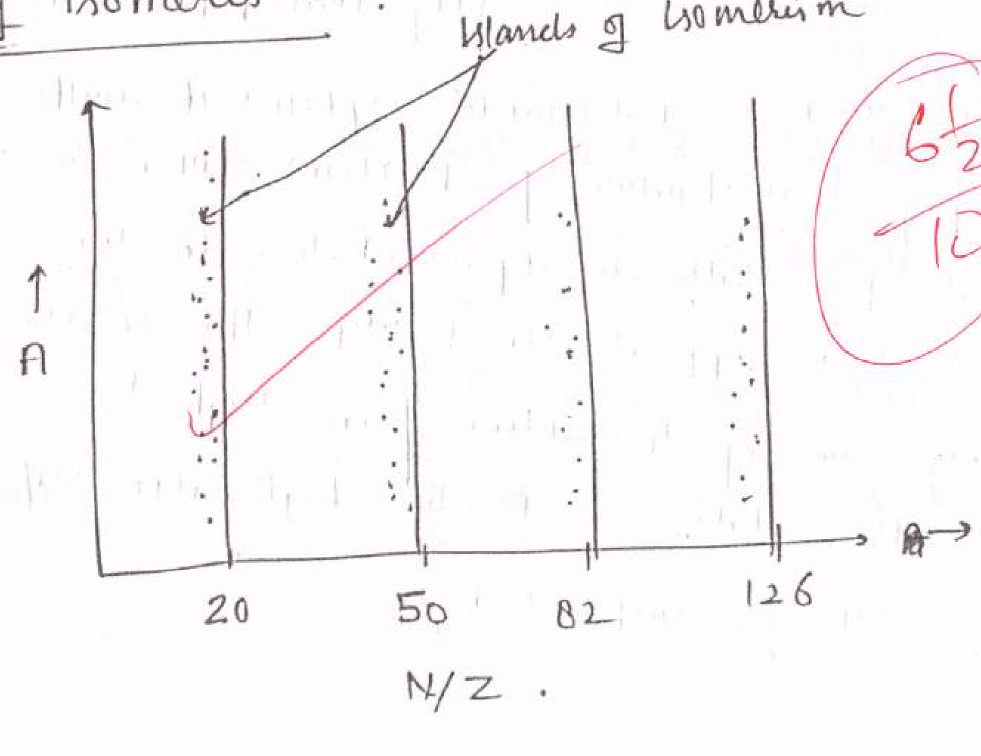


Fig. Isomerism explained with shell model.

These are found near the closed shell of Nuclei constituents i.e. magic No. (20, 28, 50, 82, 126) known as Islands of Isomerism.



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Ans1(b) (i) $^{13}_{6}C$: 6 protons + 7 Neutrons

→ give zero $\vec{I}$ because of pairing.

7 Neutrons as per shell model (configuration)

$7n = 1s^2 \, 1p^4_{3/2} \, \underline{\underline{1p^1_{1/2}}}$

so angular momentum $= \frac{1}{2}$

parity $= (-1)^l = (-1)^1 = (-1)$

so $\boxed{\text{state} = \left(\frac{1}{2}\right)^-}$

(ii) $^{16}_{7}N$:

7 protons : $1s^2 \, 1p^4_{3/2} \, \underline{\underline{1p^1_{1/2}}} \rightarrow \left(\frac{1}{2}\right)^- = j_1^{\pi}$

9 Neutrons: $1s^2 \, 1p^6 \, 1d^1_{5/2} \rightarrow \left(\frac{5}{2}\right)^+ = j_2^{\pi}$

So using <u>Nordheim's Rule</u>

$l_1 + l_2 + s_1 + s_2 = \frac{5}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} = 4 = \text{even}$

So net angular momentum $= |j_1 - j_2|$

$= \frac{5}{2} - \frac{1}{2}$

$= 2.$

parity $= (-1)^{l_1+l_2} = (-1)^{-1+2} = (-1)$

so $\left({}^{16}_{7}N\right)$ $\boxed{j = 2^-}$

(iii) ${}^{17}_{8}O_9$:

8 protons: '0' angular momentum.

9 Neutrons: $1s^2 1p^6 1d^1_{5/2}$

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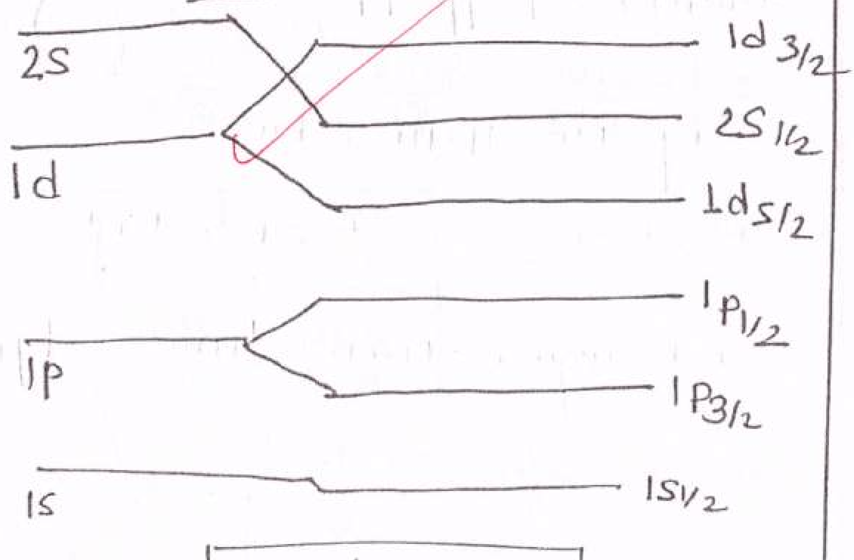
so $\boxed{j = \left(\frac{5}{2}\right)^+}$

Shell model predicts angular momentum and parity very accurately except some exceptions like ${}^{23}_{11}Na$.



shell structure

Ans 1(C) Semi-Empirical mass formula was given by Weizsacker for accurate binding energy calculation.

It is given by:

$$B.E. = a_v A - a_s A^{2/3} - \frac{a_c Z(Z-1)}{A^{1/3}} - a_a \frac{(A-2Z)^2}{A}$$
$$(\pm, 0)\, a_p A^{-3/4}$$

where constants have their normal meaning.

So for most stable isobar:

$$\left.\frac{BE}{Z}\right|_{Z=Z_0} = 0 \Rightarrow -\frac{a_c}{A^{1/3}}\left[2Z-1\right] - \frac{a_a}{A}\left[2(A-2Z)(-2)\right] = 0.$$

$$\Rightarrow -(2Z-1)\frac{a_c}{A^{1/3}} + \frac{a_a}{A}\left[4(A-2Z)\right] = 0$$

$$Z_0\left[-\frac{2a_c}{A^{1/3}} - \frac{8a_a}{A}\right] + \frac{a_c}{A^{1/3}} + 4a_a = 0$$

$$\text{or} \quad Z_0 = \left(\frac{\frac{a_c}{A^{1/3}} + 4a_a}{\frac{2a_c}{A^{1/3}} + \frac{8a_a}{A}}\right)$$

Putting values for $A = 43$.

$$Z_0 = \left[\frac{0.583}{43^{1/3}} + 4\times 19.3\right]\left[\frac{2\times 0.583}{43^{1/3}} + \frac{8\times 19.3}{43}\right]^{-1}$$

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$Z_0 = 19.69$

$\boxed{Z_0 \sim 20}$

So stable Isobar = $\boxed{{}^{43}_{20}Ca}$

$6\frac{1}{2}$ / 10

Ans 1(d) For proton (uud): $\mu_p = \left[\frac{4}{3}\mu_u - \frac{1}{3}\mu_d\right]$ —①

For neutron (ddu): $\mu_n = \left[\frac{4}{3}\mu_d - \frac{1}{3}\mu_u\right]$ —②

Now for quarks:

u: $\mu_u = \left(+\frac{2}{3}e\right)\frac{\hbar}{2m_u}$ (with $S = \frac{1}{2}$).

$\boxed{\mu_u = \frac{e\hbar}{3m_u}}$ (m_u = mass of up quark)

d: $\mu_d = \left(-\frac{1}{3}\right)\cdot\frac{e\hbar}{2m_d} = \frac{-e\hbar}{6m_d}$ (m_d = mass of down quark)

putting values in eqn – ① and we get

$\mu_p = \frac{1}{3}\left[4\times\frac{e\hbar}{3m_u} + \frac{1}{\cancel{8}}\frac{e\hbar}{6m_d}\right] = \frac{1}{3}\frac{e\hbar}{m}\left[\frac{4}{3} + \frac{1}{6}\right]$

$\boxed{\mu_p = \frac{e\hbar}{2m}}$ —③ ($m = m_u = m_d$ given)

Illy: $\mu_n = \frac{1}{3}\left[4 \times \frac{(-e\hbar)}{6 m_d} - \frac{e\hbar}{3 m_u}\right]$

$$= \frac{e\hbar}{3m}\left[-\frac{4}{6} - \frac{1}{3}\right]$$

$$\mu_n = -\frac{e\hbar}{3m} \longrightarrow (4)$$

dividing eq^n- (3) by eq^n- (4)

$$\frac{\mu_p}{\mu_n} = \frac{(e\hbar/2m)}{(-e\hbar/3m)}$$

or $\boxed{\frac{\mu_n}{\mu_p} = -\frac{2}{3}}$ ANSWER

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Ans 1(e) Magnetic moment of a nucleus is because of the unpaired nucleons which give rise to angular momentum (I).

Nucleus with one nucleon outside core

$\mu = \mu_l + \mu_s$ (orbital + spin)

$$= \frac{e}{2m}\left[|\vec{L}|\, g_l \cos(\vec{L},\vec{J}) + |\vec{S}| \cos(\vec{S},\vec{J})\cdot g_s\right]$$

$$= \frac{e\hbar}{2m}\left[g_l \sqrt{l(l+1)}\left\{\frac{j(j+1)+l(l+1)-s(s+1)}{2\sqrt{j(j+1)}}\right\} + g_s\sqrt{(s)(s+1)}\right.$$

$$\ldots \cdot \frac{j(j+1)+s(s+1)-l(l+1)}{2\sqrt{j(j+1)}}\Big]$$

multiplying by $\sqrt{j(j+1)}$ in numerator & denominator and summing up we get:

$$\boxed{\mu = g_j \mu_N \sqrt{j(j+1)}}$$

where $g_j = \left[g_l \frac{j(j+1)+l(l+1)-s(s+1)}{2j(j+1)} + g_s \frac{j(j+1)+s(s+1)-l(l+1)}{2j(j+1)} \right]$

Now for z-component:

$$\boxed{\mu_z = g_j \mu_N j} \quad (\text{when } j = (m_j)_{max}).$$

for $j = l+\frac{1}{2}$; $\mu_z = \left[\left(j+\frac{1}{2}\right)g_l + \frac{g_s}{2}\right]\mu_N$

for $j = l-\frac{1}{2}$ $\quad \mu_z = \left[\left(j+\frac{3}{2}\right)g_l - \frac{g_s}{2}\right]\left(\frac{j}{j+1}\right)\mu_N$.

for ${}^{17}_{8}O_9$: 9 Neutron $= 1s^2 1p^6 1d^1_{5/2}$

9 neutrons: 1 in outside shell of core shell.

So $\mu_z = ?$ $\quad g_l = 0$ (Neutron)

$\mu_z \Rightarrow j = l+\frac{1}{2}$ or $\frac{5}{2}$ so $\mu_z = -\frac{g_s}{2}\mu_N$

$\boxed{\mu_z = -1.91\mu_N}$ ANSWER.

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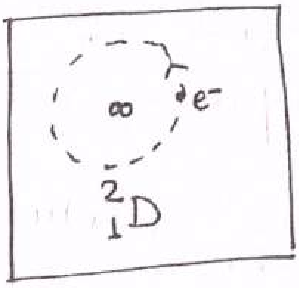
UNE ?

Ans 2 (a) Deuteron is simplest molecule with 1 proton and one neutron that is stable yet loosely bound with binding energy of around 2.25 MeV.



Study of Deuteron provides following information about nuclear forces:

(1) SPIN DEPENDENT — Nuclear forces are spin dependent because for singlet state (S=0), no bound state of deuteron exits which shows that Nuclear force is spin dependent.

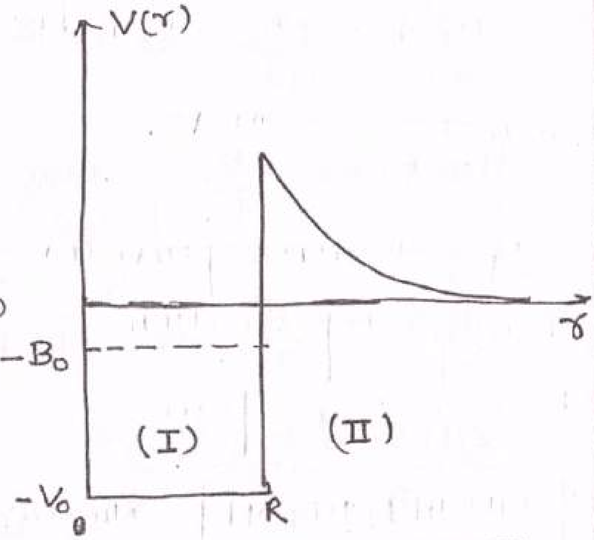
[2] Charge independence: ^2_1D has proton (Q=+e) and neutron (Q=0) and the nuclear force is not affected by either charge.

[3] SHORT range — Deuteron nucleus is loosely bound within range of 10^{-15} m or one fermi showing Nuclear forces are

short range.

[4] Non-central force: Mixed of $l=0$ & $l=2$ States.

(ii) Given: Nuclear attractive potential $V = 30$ MeV



Potential curve for ^2_1D

Schrödinger equation for ^2_1D Nucleus wavefunction ψ.

$$\frac{d^2u}{dr^2} + \left[\frac{2\mu}{\hbar^2}(E-V)u\right]r = 0$$

This is for $l=0$;

where $u = rR(r)$

Here $E = -B_0$ (Binding energy of ^2_1D)

$V = -V_0$ (potential)

or $\frac{d^2u}{dr^2} + \frac{2\mu}{\hbar^2}(V_0 - B_0)u = 0$. for $r<R$

$u'' + k_1 u = 0$ $\boxed{u_1 = A\sin k_1 r}$ (after Boundary conditions)

and for region-(ii) $\Rightarrow V_0 = 0$, $E = -B_0$

$\boxed{u_2 = Ce^{-k_2 r}}$ with $k_2 = \frac{2\mu B_0}{\hbar^2}$

Boundary conditions show that:

$\psi_r = R(r) = \frac{u_1}{r} = \frac{A\sin k_1 r}{r} = 0$ at $r = 0$.

Now after solving for B.Cs :

(i) $\psi_{r_1}(r = R) = \psi_{r_2}(r = R)$ and

(ii) $\frac{d\psi_1}{dr} = \frac{d\psi_2}{dr}$ at $r = R$ we get

$\boxed{\tan k_1 R = -\frac{k_1}{k_2}}$ and as $B_0 \ll V_0$ $\left(k_1 R = \frac{\pi}{2} + \frac{2k_2 R}{\pi}\right)$

so $k_1 R \rightarrow \frac{\pi}{2}$ or $k_1^2 R^2 = \frac{\pi^2}{4}$

$$\frac{2\mu(V_0 - B_0)}{\hbar^2} \cdot R^2 = \frac{\pi^2}{4}$$

or $$\frac{2\mu V_0}{\hbar^2} \times \frac{4}{\pi^2} = R^{-2}$$

or $$R^2 = \frac{\pi^2 \hbar^2}{8\mu V_0}$$

$$R = \frac{\pi \hbar}{\sqrt{8\mu V_0}} = \frac{3.14 \times 1.05 \times 10^{-34}}{\sqrt{8 \times \frac{1.67 \times 10^{-27}}{2} \times 30 \times 1.6 \times 10^{-13}}}$$

$$= 1.79 \times 10^{-15}\ m$$

$\boxed{R \cong 1.8 \text{ Fermi}}$

Hence the size of ${}_1^2D$ Nucleus is around

$\boxed{1.8 \text{ Fermi}}$

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Ans 2(b) Stability of Magic Numbers: Shell model given by Mayer using the spin-orbit coupling of nucleus angular momentum explains the high stability of magic No's - 2, 8, 20, 28, 50, 82 & 126 :-

(1) Magic Nos are Neutron or proton No is a nucleus which are closed shell (or fully occupied).

(2) This leads to their extra-stability as for closed shells the stability due to high binding energy increases.

(3) Example: ${}^{4}_{2}He$: $\left.\begin{array}{l} 2p = 1s^2 \\ 2n - 1s^2 \end{array}\right]$ fully closed

or ${}_{28}Ni$: $28p = 1s^2 1p^6 1d^{10} 2s^2 1f^8_{7/2}$ (fully closed).

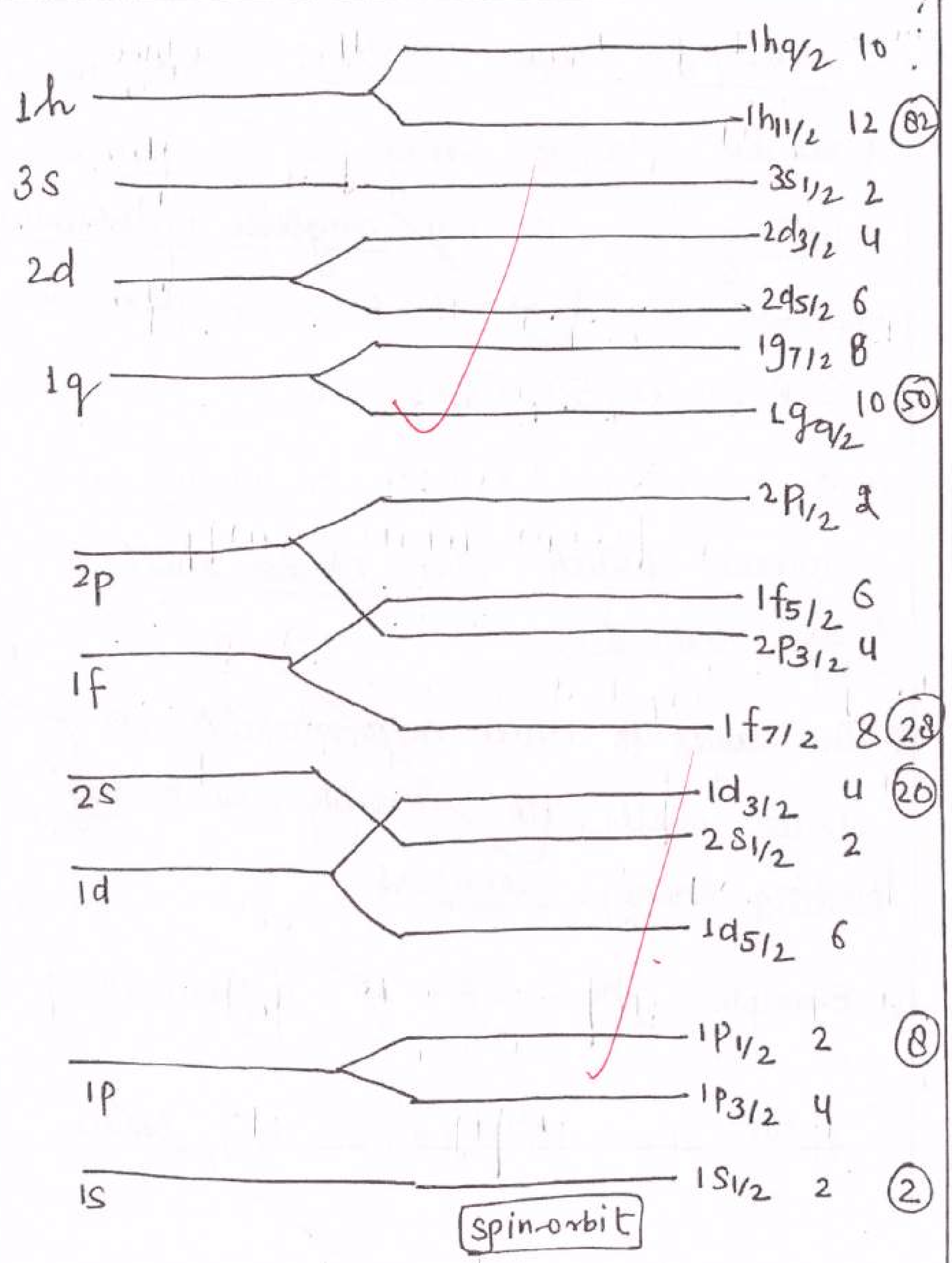

Fig. Shell Model of Nucleus: Magic Nos

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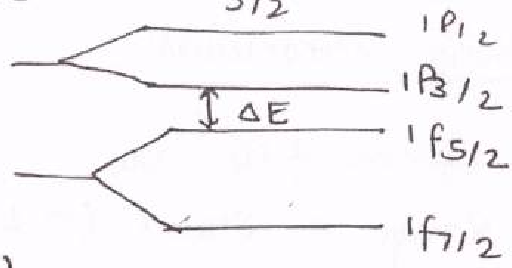
Given: ΔE for $1f_{5/2}$ and $1p_{3/2}$ = 8 MeV

$^{207}_{82}Pb$



$$\Delta E_{S-O} = -\alpha(\vec{S}\cdot\vec{L})$$

$$= -\alpha\left(\frac{I^2+L^2-S^2}{2}\right) = \alpha(L^2+S^2-I^2)$$

$$= -\alpha\,\frac{[I(I+1)-L(L+1)-S(S+1)]}{2}\hbar^2$$

E_1 ($j = 5/2$: $'f_{5/2}$) $l = 3$, $s = \frac{1}{2}$.

$$E_1 = -\frac{\alpha\hbar^2}{2}\left[\frac{5}{2}\times\frac{7}{2} - 3\times 4 - \frac{1}{2}\times\frac{3}{2}\right]$$

$$\boxed{E_1 = 2\alpha\hbar^2}$$

See the soln.

and $E_2 = -\frac{\alpha\hbar^2}{2}\left[\frac{3}{2}\times\frac{5}{2} - 1\times 2 - \frac{3}{4}\right]$

$j_2 = 3/2$
$l_2 = 1$
$s_2 = \frac{1}{2}$

$$= -\frac{\alpha\hbar^2}{2}[1] = -\frac{\alpha\hbar^2}{2}.$$

$$\boxed{E_2 = -\frac{\alpha\hbar^2}{2}}$$

$$\frac{13}{20}$$

$$E_2 - E_1 = 8\text{ MeV} = \hbar^2\left[2+\frac{1}{2}\right]\alpha$$

$$= \alpha\hbar^2\frac{5}{2}.$$

$$\boxed{\alpha = 8\text{ MeV}\times\frac{2}{5\hbar^2}}$$

~2 MeV

Ans 2(c) Particle interactions -

① Strong interactions:

(i) Lifetime = 10^{-23} sec

(ii) Range = Short (~ 1 fermi)

(iii) Particles exchanged = π-Mesons

(iv) Relative strength = Very high (100 times EM)

(v) Characteristic features -

(A) Responsible for nuclear force

(B) Interact via gluons

(C) Strangeness and other conservation laws are obeyed.

eg

$$\boxed{\pi^+ + p^+ \longrightarrow K^+ + \Sigma^+}$$

② EM Interactions

(i) Δt = very low

(ii) Range = Infinite (∵ of zero photon mass)

(iii) Interact via PHOTON exchange

(iv) strength is $\frac{1}{100}$ of strong intn.

(v) characteristic features - Responsible for charged interactions.

(vi) Massless photon is responsible.

eq $\boxed{e^+ + e^- \longrightarrow \gamma + \gamma}$

(iii) <u>weak interaction</u> -

- Lifetime = 10^{-10} sec
- Range = very very short (10^{-17} m)
- Particle exchanged = $W^{\pm}$ and Z^0 Bosons
  ($80.4 GeV/c^2$) ($90 GeV/c^2$)
- Strength is very weak.
- Responsible for strange particle decays such as $\Sigma, \Lambda, \Omega^-$ etc.
- β-decay is example of this interaction

eq $\boxed{n \longrightarrow p + {}_{-1}^{0}\beta + \bar{\nu}}$

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(iv) <u>Gravitational Interaction</u>: Exchange particle <u>graviton (spin = 2)</u> is responsible for this int^n with infinite range ($m_\gamma \cong 0$). It has relative strength at its lowest.

Relative strength.

$\boxed{\text{strong} > \text{EM} > \text{weak} > \text{Gravitational}}$

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Question :3

Ans 3(a) Quark: Up(U) :

$I_z = 1/2$, $S = 0$

$Y = \cancel{B} B + s + c + b + t$.

$= \frac{1}{3} + 0 = \frac{1}{3}$

$\boxed{Y = \frac{1}{3}}$

Down Quark d $I_z = -\frac{1}{2}$.

$S = 0$

$Y = \frac{1}{3}$

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Strange quark (S): $I_z = 0$

$S = -1$

$Y = B + S + C + b + t$

$= \frac{1}{3} + (-1) = \frac{2}{3}$

$\boxed{Y = \frac{2}{3}}$

PROTON has two upquarks and one down quark

$\boxed{P = uud}$

$Q = +1$

$B = 3 \times \frac{1}{3} = 1$

$S = 0$.

$I_z = \frac{1}{2}$

Neutron: $\boxed{n = udd}$: $Q = 0$

$S = 0$

$B = 1$

$I_z = -\frac{1}{2}$.

Ans5(b) $\boxed{\text{Neutrino detection}}$ – Cowan and Reins experimented in the following manner:

Tank filled with H_2O & $CdCl_2$ was placed near a nuclear reactor which provided Neutrinos ($\nu_e, \bar{\nu}_e$).

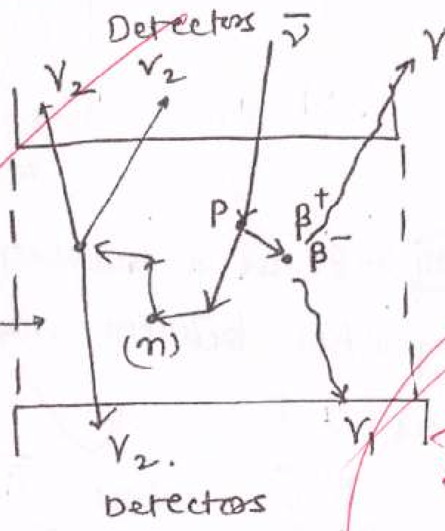

Fig. Experimental setup

Now,

$$\because \boxed{p + \bar{\nu}_e \longrightarrow n + {}^{0}_{+1}\beta}$$

The $\bar{\nu}_e$ (anti-neutrino) knocked out proton into neutron and β^+ particles.

→ β^+ annihilated with β^- leading to 2γ rays detected. $\boxed{\beta^+ + \beta^- \longrightarrow \gamma + \gamma}$

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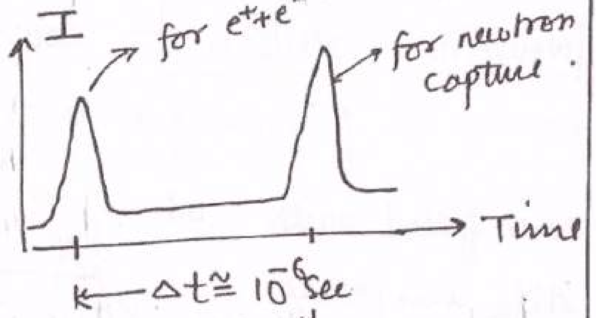
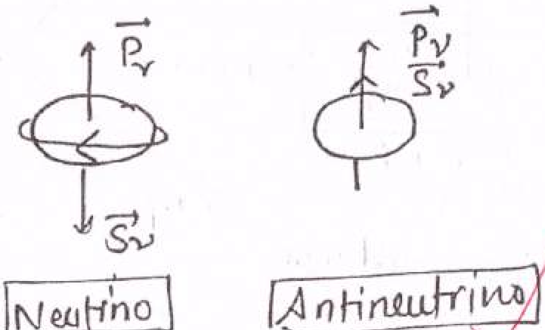
Neutron after few microsecond gets captured by $CdCl_2$ and 3 γ-photons are emitted, detected by detectors.

This confirms the presence and detection of anti-neutrinos.



(Helicity) - It is a property of neutrinos which distinguishes between neutrinos and anti-neutrino.

$H = \frac{\vec{P}\cdot\vec{S}}{|\vec{P}||\vec{S}|}$ where $\vec{P}$ = Momentum
$\vec{S}$ = Spin vector



Helicity $= \pm 1$ provides that the independence of frame of reference.

Ans3(c) ① strange particles are produced with strong reactions but they are decayed through the weak interactions (10^{-10} sec)

Ex $p^+ + \pi^+ \longrightarrow K^+ + \Sigma^+$

- Both strange
- Produced in pair

② Paired production: They are always produced in association and not alone.

Gellmann - Nishijima relation

Charge (Q) = $(\text{Isospin})_z + \frac{\text{Hypercharge}(Y)}{2}$

$$\boxed{Q = I_3 + \frac{Y}{2}}$$

where $Y = B + S + c + b + t$

B → Baryon No.; S → Strangeness; c → Charm; b → bottom; t → top

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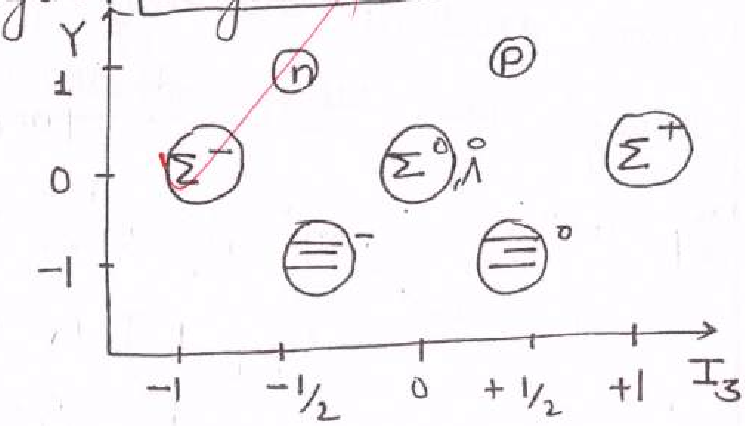
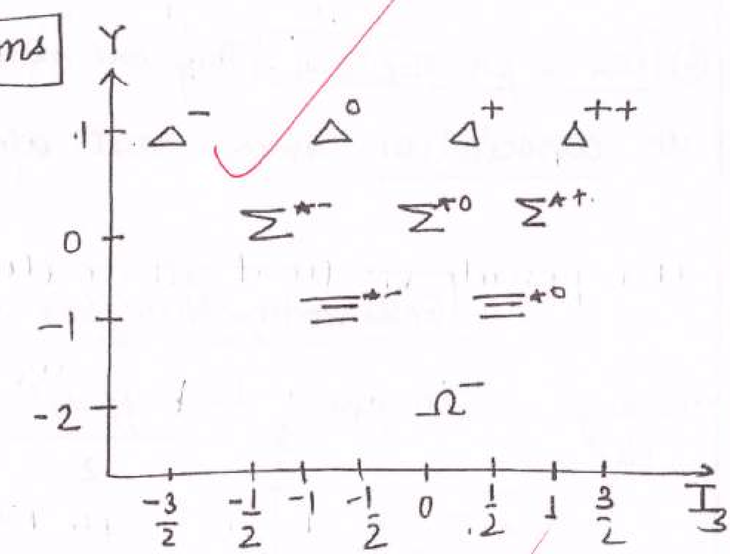
Classification of particles

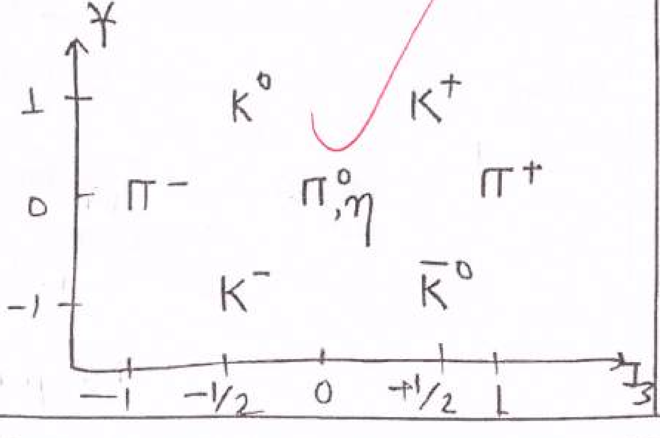
① spin $\frac{1}{2}$ Baryons : Baryon Octet



② Spin $\frac{3}{2}$ Baryons



③ Spin 0 Mesons



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Ans 3(d)

(i) $n \longrightarrow p + e^- + \bar{\nu}_e$

0   0   1   1

lepton No. Not conserved $\Delta L = +2$

So $\boxed{\text{Not permitted}}$

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(ii) $p + \pi^- \longrightarrow \pi^- + \Sigma^+$

	p	π^-	π^-	Σ^+	
Q	1	−1	−1	1	(✓)
B	1	0	0	1	(✓)
S	0	0	0	−1	$\Delta S = -1$
I	$\frac{1}{2}$	1	1	1	$\Delta I = \frac{1}{2}$ Not Conserved
I_3	$\frac{1}{2}$	−1	−1	1	$\Delta I_3 = \frac{1}{2}$. Not Conserved

Neither by strong nor EM, Forbidden

~~It is possible via weak interaction~~

(iii) $\Sigma^0 \longrightarrow \lambda^0 + \gamma$

Q   0   0   0

Spin $\frac{1}{2}$ $\frac{1}{2} + 1$ ~~(spin not conserved so not possible)~~ Conserved

(iv) $K^- + p \longrightarrow n + \lambda^0$

Q   −1   1   0   0   (✓)

B   0   1   1   1 $\Delta B \neq 0$ so not possible due to Baryon No. violation.

Que 4 :

(a) Grand Unified Theory Schemes

(1) Electroweak theory – unifies electromagnetic and weak interactions with explaining for very high temperature the force of quanta of both interactions (photon and $W^{\pm}, Z^{0}$ bosons) were manifestation of a single force. It is very much possible at very high temperature to have symmetry but as the temp cooled down, symmetry was broken and resulted into two different forces with different force of quantas.

(2) Grand Unified theory – It predicts the existence of (LEPTO-QUARK) and unifies the 3 interactions – Strong with EM-weak interaction.

– As the spins of gauge bosons is '1' it was possible for very high energy (in GeV) that there existed only one single force with quanta as Lepto-quark.

This theory leads to following predictions

① Existence of very large magnetic monopoles.

② Leptonic decay of Baryons leading to Baryon number violation.

$$p \longrightarrow \pi^0 + e^+ \quad (\Delta B \neq 0).$$

The Grand Unified theory can be explained using the very highly symmetric Group SU(5) which will provide for the high symmetry.

— As the universe cooled down, the symmetry was broken and resulted in

(i) Strong reaction

(ii) Electroweak interaction — further cooling led to — weak interaction
— Electromagnetic interaction.

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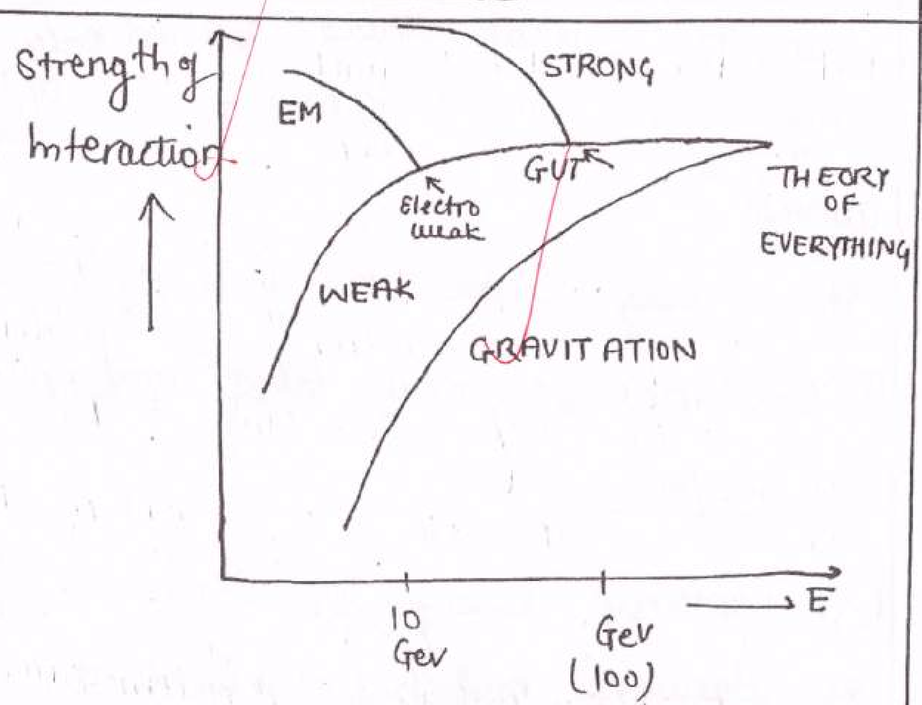


Experimental verifiable ~~scheme~~ predictions

① Existence of Higgs bosons by Standard model which provides masses to particles with Mass = few TeV/c^2.
(detected in 2012 in CERN LHC)

② Existence of Leptoquark.

③ Magnetic monopoles.

④ Existence of 6 Coloured gluons and 6 antiparticle of these with charges $-\frac{1}{3}|e|$ $[Y_R, Y_G, Y_B]$ and $-\frac{4}{3}[|e| (X_R, X_G, X_B)]$

which when combined with (γ, $W^{\pm}$, Z°, 8 gluons) give 24 gluons.

All the schemes predict the unification of 3 interactions. Yet gravity or gravitational force is yet to be reconciled with other 3. Problems being

(i) Spin of gluon = 2

(ii) Very weak interaction at quantum level

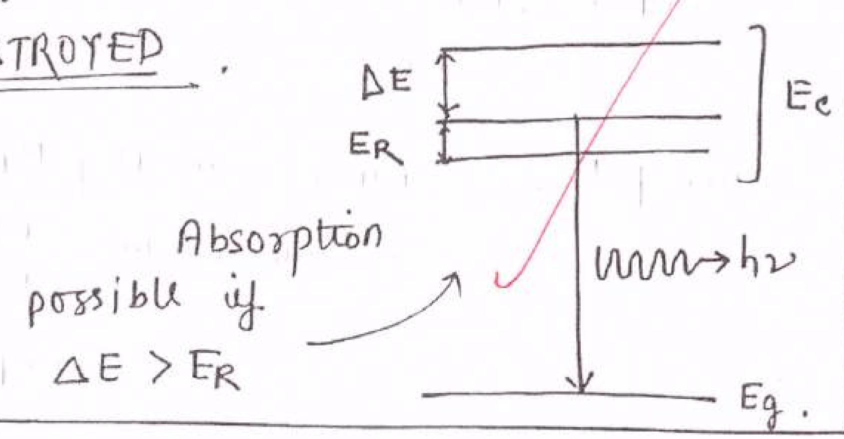
(iii) Quantum gravity is hard to deal with.

If all 4 forces are unified then it will be the Theory of Everything.

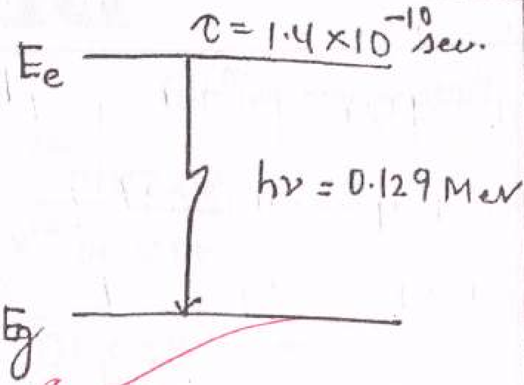
Ans 4(b) Mossbauer effect is recoilless emission and absorption of γ-Ray photons by nuclei. It helps understand the very fine γ-Ray energies (precise) and probing of nucleus with these.

Mossbauer spectra: when there is low lying excited state of nucleus, the transition between excited and ground state releases γ-ray photon which also causes nucleus to recoil and thus reducing the γ-Energy.

If the recoil energy E_R of nucleus is more than the uncertainity in energy for lifetime (τ) then the absorption is DESTROYED.



For ${}_{77}I^{191}$



The excited state's time period is

$\tau = 1.4\times10^{-10}$ sec

Now the uncertainty in Energy associated with it

$$\Delta E \cong \frac{\hbar}{\tau} = \frac{1.05\times10^{-34}}{1.4\times10^{-10}} = 7.5\times10^{-25}\,J.$$

$$= \frac{7.5\times10^{-25}}{1.6\times10^{-19}}$$

$$\boxed{\Delta E = 4.69\times10^{-6}\ eV}$$

Now for $E_r = 0.129$ MeV the Recoil energy of nucleus $E_R = \frac{E_r^2}{2mc^2}$ where m = mass of Nucleus

so $$E_R = \frac{(0.129\times1.6\times10^{-13})^2}{2\times M\times(3\times10^8)^2} \rightarrow ①$$

Calculation of M: $\because$ 191 gm is weight of N_A Nucleus

so for 1 nucleus $M = \frac{191}{N_A}$

$= 3.17\times10^{-25}$ Kg.

Putting in eq^n-①

$$E_R = \frac{9.25\times10^{-46}}{3.17\times10^{-25}\times1.6\times10^{-19}} \text{ eV}$$

$$= 1.82\times10^{-2} \text{ eV}$$

$$\boxed{E_R = 1.8\times10^{-2} \text{ eV}}$$

Now it is clear that E_R is more than ΔE by order of 10^4 eV.

Hence the resonant absorption to first excited state ($\tau = 1.4\times10^{-10}$ s) will be impossible.

To have resonant absorption we need no recoil of nucleus. It can be done via –

① Doppler broadening of energy levels

② crystal lattice solids embedding.

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Ans 4(c)

α-decay: $_{Z}^{A}X \longrightarrow {}_{Z-2}^{A-4}Y + {}_{2}^{4}He \;(\alpha)$

Given $A = 240$

$Q = 5.26$ MeV

To find $K_\alpha = ?$

$$\because \quad Q = K_\alpha\left(\frac{A}{A-4}\right)$$

$$\Rightarrow \quad K_\alpha = \left(\frac{A-4}{A}\right)Q$$

$$= \frac{240-4}{240} \times 5.26 \text{ MeV}$$

$$\boxed{K_\alpha = 5.17 \text{ MeV}}$$

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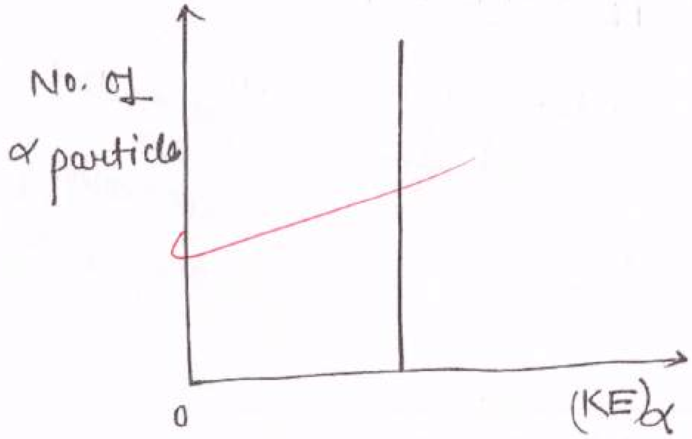
α-decay spectrum of energy will be a line-spectrum with K = 5.17 MeV



QUESTION-5

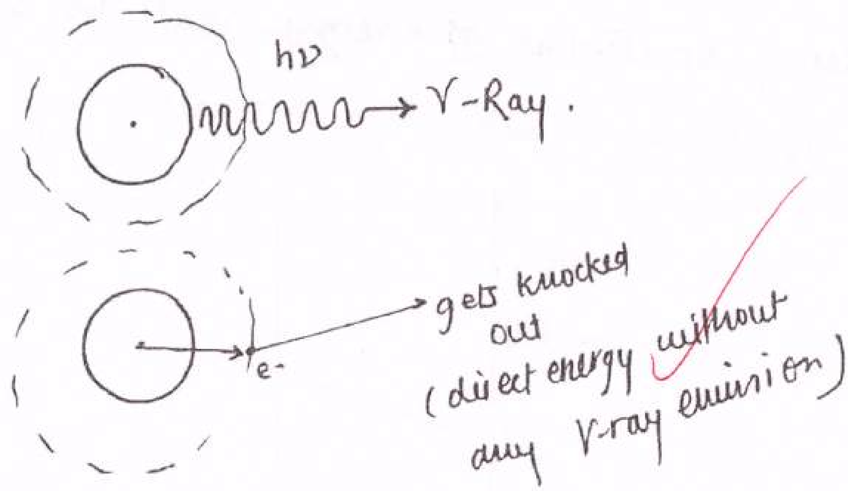
Ans 5(a) $\boxed{\gamma\text{-decay}}$: When excited nucleus decays into ground state by emitting γ-ray photon.

Selection rule: $|\vec{I}_i - \vec{I}_f| \leq \vec{L} \leq \vec{I}_i + \vec{I}_f$

or γ-photon must carry at least '1' angular momentum.

$\boxed{\text{Internal conversion}}$ - Nucleus de-excites by transferring the energy to one of the electrons in electronic shells.

There is NO change in spin of Nucleus during the process.



प्रश्न संख्या (Question No.)

DIAS

इस भाग में कुछ न लिखें (Don't write anything in this part.)

Internal conversion depends on –

(i) the overlapping wavefunctions of electron and Nucleus.

(ii) The energy of de-excitation must be more than binding energy of electron.

$\boxed{h\nu > B.E.}$

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Ans5(b)

Binding energy as per the semi-empirical mass formula depends on various terms.

The main reasons for constancy between $30 \leq A \leq 170$ are –

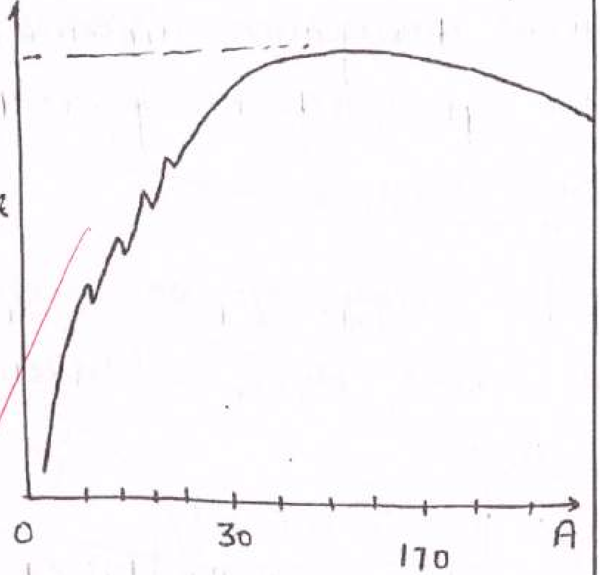

Fig. B.E/A curve

(1) Saturation: Nucleons interact only with the nearest nucleons and not with all of nucleons. Hence the B.E/A saturates leading to a plateau b/w 30 and 170.

(2) As the proton no. rises so does neutron no. and because of strong Nuclear force the coulomb repulsion is countered and neutrons become more than protons leading to balance.

(3) Between $30 \leq A \leq 170$ the coulomb repulsion is not much but beyond this it starts to decrease the B.E/A.

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Ans 5(C) Weizsacker gave semi-empirical mass formula to explain Binding energy of nuclei.

Formula

$$B.E. = a_v A - a_s A^{2/3} - a_c \frac{Z(Z-1)}{A^{1/3}} - a_a \frac{(A-2Z)^2}{A} (\pm, 0)$$

$$a_p A^{-3/4}$$

with this

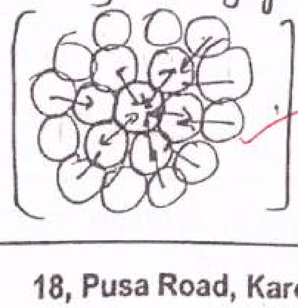
Mass of any nucleus: $\boxed{M(Z,A) = Zm_p + Zm_n - \frac{B.E.}{c^2}}$

(i) Volume term: $(B_v = a_v A)$

Because of nucleon strong nuclear forces these are tied strongly increasing binding energy of nucleus just like in liquid drop.

(ii) Surface term: surface nucleons are less tied and loose in comparison to inner ones and they decrease the volume Binding energy.

$\boxed{B_S = -a_s A^{2/3}}$



(as it is proportional to surface area).

(iii) Asymmetric energy: Arises due to asymmetry in proton and neutron number.
Increased neutron no. counters the coulomb energy repulsion.
Thus the B.energy is decreased.

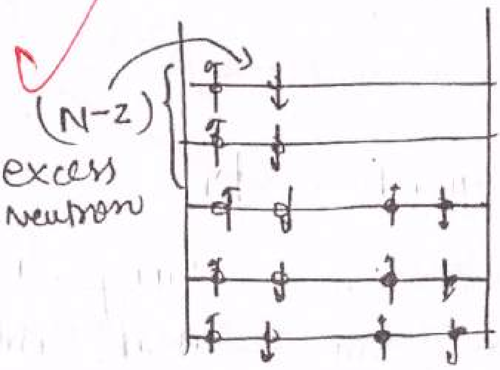
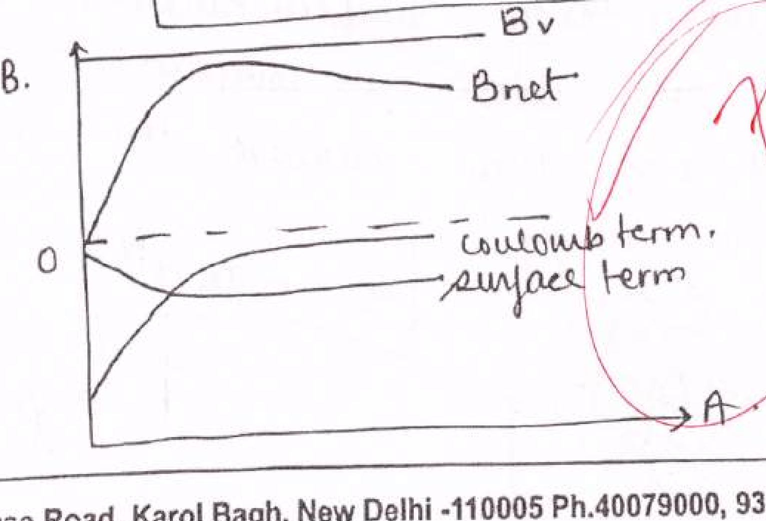


Fig. Asymmetric term

(iv) Coulomb energy –
Protons coulomb repulsion causes the B.E. to decrease further.

$$E_c = +\frac{3}{5}\frac{Z_1 Z_2 e^2}{4\pi\epsilon_0 r}$$

for protons: $\boxed{E_c = \frac{3}{5}\frac{Z(Z-1)e^2}{4\pi\epsilon_0 R} = -a_c\frac{Z(Z-1)}{A^{1/3}}}$



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Ans 5(d) (i) Isotopic Spin (I):

conserved in

① Strong Interaction

②

Violated in

① weak Interaction

② Electromagnetic "

(ii) Hypercharge (Y)

conserved in – Strong, EM interaction

violated in – weak interaction

(iii) Lepton Number(L) is conserved in all 3 interactions

(iv) charge conjugation is conserved in Strong and EM but violated in weak interactions

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Ans 5(e) Quark is basic constituent unit of Hadrons (Baryons and Mesons).

(i) π^+ : $u\bar{d}$

(ii) K^+ : $u\bar{s}$ (strange particle $S=+1$)

(iii) Δ^{++} : uuu

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(iv) Σ^0 : uds (strange particle $S=-1$)

(v) Ω^- : sss ($s=-3$).